§1.3 multiple species interactions (Continuous Case) with Lotka-Volterra Type.

1. Predator-Prey system Lotka-Volterra type

Let $x(t)$ be the population density of prey at time t, $y(t)$ be the population density of predator, Assume prey grows logistically in the absence of predation.

$$(1)\quad \begin{cases} \dfrac{dx}{dt} = rx\left(1-\dfrac{x}{K}\right) - \alpha x(t)y(t) \\ \dfrac{dy}{dt} = \beta x(t)y(t) - d y(t) \end{cases}$$

Assume prey grows exponentially in the absence of predation

$$(2)\quad \begin{cases} \dfrac{dx}{dt} = ax - \alpha x y \\ \dfrac{dy}{dt} = \beta x y - d y \\ x(0) > 0,\ y(0) > 0 \end{cases}$$

Per-Duet

Two-species

2. Competition system Lotka-Volterra Type

Let $x(t)$, $y(t)$ be two competing species.

In the absence of competition ~~of compet~~, the species $x_1(t)$, $x_2(t)$ grows logistically with carrying capacity K_1, K_2 respectly. Then the model takes the form

$$
\begin{cases}
\dfrac{dx_1}{dt} = r_1 x_1\left(1-\dfrac{x_1}{K_1}\right) - \alpha_1 x_1 x_2 \\
\cancel{\dfrac{dy}{dt} = r_2 y(1-} \\
\dfrac{dx_2}{dt} = r_2 x_2\left(1-\dfrac{x_2}{K_2}\right) - \alpha_2 x_1 x_2
\end{cases}
$$

3. Mutualism (共生)

$$\frac{dx_1}{dt} = r_1 x_1\left(1-\frac{x_1}{K_1}\right) + \alpha_1 x_1 x_2$$

$$\frac{dx_2}{dt} = r_2 x_2\left(1-\frac{x_2}{K_2}\right) + \alpha_2 x_1 x_2$$

4. Food chain model

X prey

Y predator

Z Top predator

$$\begin{cases} \frac{dx}{dt} = rx\left(1-\frac{x}{K}\right) - \alpha_1 xy \\ \frac{dy}{dt} = \beta_1 xy - d_1 y - \alpha_2 yz \\ \frac{dz}{dt} = \beta_2 yz - d_2 z \end{cases}$$

$x(0) > 0, \quad y(0) > 0, \quad z(0) > 0$